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Weighted networks of scientific communication: the measurement and topological role of weight

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Abstract

In order to take the weight of connection into consideration and to find a natural measurement of weight, we have collected papers in Econophysics and constructed a network of scientific communication to integrate idea transportation among econophysicists by collaboration, citation and personal discussion. Some basic statistics such as weight per degree are discussed in Fan et al. J. Mod. Phys. B (17–19) (2004) 2505. In this paper, by including the papers published recently, further statistical results for the network are reported. Clustering coefficient of weighted networks is introduced and empirically studied in this network. We also compare the typical statistics on this network under different weight assignments, including random and inverse weight. The conclusion from weight-redistributed network is helpful to the investigation of the topological role of weight.

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Keywords: Weighted networks; Networks of scientists; Clustering coefficient; Random weight

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1. Introduction

Recently, many researchers in different fields use the topological properties and evolutionary processes of complex networks to describe the relationships and collective behaviors in their own fields [1,2]. This methodology, which is so-called network analysis, often leads to discoveries. Also new analysis methods and new topology properties are proposed by this approach. A network is a set of vertices and a set of edges which represent the relationship between any two vertices. Just because of its simplicity of this description, network can be used in so many different subjects (see Ref. [1] and its references), including linguistics, collaboration of movie actors and scientists, human sexual contacts, disease propagation and controls, community structures, information networks, and food webs.

However, a single line representing the existence of the relation will be a limitation when it is used to describe relations having more than one level. For instance, in the network of scientists, both collaboration and citation are the ways of idea transportation but with different contributions. When we analyze this transportation as a whole, we have to use different weights to measure these different contributions. Also, even for the same level interaction, such as collaboration, not only the existence of connection but the times of collaboration is valuable information. So to fully characterize the interactions in real networks, weight of links should be taken into account. In fact, there are already many works on weighted networks, including empirical studies and evolutionary models [3–6].

The way to measure the weight for weighted networks has been introduced differently in several types by some authors. First type, converting some quantities in non-weighted network into the weight of edge. In Ref. [7], the weight of an edge is measured by the point degrees k_i and k_j (e.g. $w_{ij} = k_i k_j$) of its two ends. Second type, in some networks, typically natural measurement of weight is already given by the phenomena and event investigated by the network. In the scientific collaboration network, the times of co-authorship are registered as the weight of link [8]. In Ref. [9], in the case of the WAN the weight w_{ij} of an edge linking airports i and j represents the number of available seats in flights between these two airports. In Ref. [10], the weight w_{ij} stands for the the total number of flights per week from airport i to airport j. The third type is in the works about modelling weighted networks. Some prior weights are introduced in [4]. In Ref. [11], the weight w_{ij} of a link l_{ij} connecting a pair of nodes (i and j) is defined as $w_{ij} = (w_i + w_j)/2$, where w_i is defined as i node's assigned number (from 1 to N) divided by N. In Refs. [12,13], the weight w is assigned to the link when it is created, which is drawn from a certain distribution $\rho(w)$. In fact, the first type of weight description should be regarded as an approach of non-weighted networks. It is helpful to discuss new properties of the non-weighted networks but without taking any more information than the non-weighted networks about the real interactions. In the second type, which is a very large class of the weighted networks, typical measurement of weight is already given by the phenomena. The investigation of such network focuses mainly on how to define and discover the topological character of the networks. In the last type, from the viewpoint of empirical study, we never know whether or not such models already acquire the real structure of weighted networks or not. In fact, giving some hints on modelling weighted network is also a part of the goals of our empirical investigation. The empirical study of weighted network without a naturally given definition of weight, is especially valuable to answer questions such as how to define a well-behavior weight, and to extract structural information from networks, and what's the role of weight according to its effects on the structure of the network.

In our work, we apply the general approach of weighted network analysis onto network-style phenomena without given measurement of weight. There are lots of such kinds of networks. For example, in our case, we try to construct and reveal the structure of the network behind the transportation of ideas in scientific community. Actually the scientific collaboration has already become an interesting subject for network researches [14,15]. But in this paper, how close two scientists are related in our network and how easily the idea transferred between them, are the phenomena we are interested in this network. This is similar to the situation that one want to construct and reveal the structure of network of underground railroad by the information about traffic, the passengers coming in and out at stations, without a map of the subway. Therefore, both the existence and the times of coauthoring (or citation, acknowledgement) are important for the network construction. And the times, for sure, implies some information about "how close and how easily" in the sense given above.

In order to extract relationship information from the times of interactions, a tanh function is used to convert the times into weight, and all the weights from coauthor, citation and acknowledgement are combined into a single weight of every edge. Tanh function starts from $\tanh(0) = 0$, and increases up to 1 when the independent variable is large enough. The times of the event, in our network, is a cumulative number. Intuitively, the more times, the closer is the relationship, and the less contribution that one new event can provide to the relationship. That means the contribution of a new event to the relationship should decrease on marginal. The reason of such a saturation effect is that, what we want to analysis is the relationship of "how closely and how easily", not the events of the transportation, although we have to start with the events and extract information from them. With the subway analogy, the railroad network of "how wide is the road between any two stations" is our object, not the traffic itself, although the only information we can make use of is about the traffic. Because of the same reason we incorporate the three weights into a single one. In the sense of idea transportation, they provide the same kind of information about "how closely and how easily", only with different contributions. Now, the next problem will be how to measure them differently by their deserved contribution. Frankly, we have no principal way to measure the "deserved" contribution. The thumbrule here is the ratio of total times of the three events, 7:2:1, which is used for their relative contributions. We have tried to reveal the effect of different relative coefficients. But the topological quantities and their distributions we have done now is not enough to describe such effects. It seems the effect of different coefficients can only be shown by some new topological quantities. Fig. 4 hints that in order to reveal such effect, we have to come to the correlation analysis.

In Ref. [16], we have constructed such a weighted network of idea transportation between scientists in Econophysics, an active field oncoming recently [17,18]. Basic statistics have been presented, including the weight per degree. In this paper, we collected most papers till July 2004 in Econophysics, and constructed this networks as a sample of weighted networks. Now we ask the questions: first, whether the distribution and property of the basic statistics change after the one year development; second, whether the way to measure the weight is significant for the structure of network and what's the effect on the structure of network if the weights on the edges are redistributed; third, what are the definitions and properties of more quantities such as cluster coefficient. The matching pattern in directed and weighted networks, the robustness of weighted networks and the topological property of weight will be discussed in later papers. The second part, the effect on network structure by changing the matching pattern between weights and edges, plays an important role in this paper. Because we think this investigation reveals the topological role of weight: does the weight affect the network significantly, and for a vertex, is there any inherent relation between its weight and its status in the network? However these questions are not fully answered in this paper yet. In this work, we randomize the relation between weights and edges with the similar idea of randomizing the connection under fixed number of edges in WS model [19]. We think this approach partially realized the idea about investigating topological role of weight.

Just because the Econophysics is hot in both Finance and Statistical Physics, our work will be of interest to econophysicists for another reason: it is about their works, and it represents the idea transportation between them.

2. Measurement of weight and basic statistical results

Recently more and more researchers in economics take up Statistical Physics to explore the dynamical and statistical properties of financial data, including time series of stock prices, exchange rates, and the size of organizations [20,21]. Meanwhile many physicists from Statistical Physics and Complexity turn to working in finance, as an important and copies research subject.

To investigate the development of such a new subfield is an interesting work itself. In our previous paper [16], we have introduced the work of paper collection and the construction of the scientific communication network. Concentrating on main topics of Econophysics, we collected papers from the corresponding journal. The basic statistical results of the network was given in Ref. [16]. It was constructed by papers published from 1992 to 4/30/2003, including 662 papers and totally 556 authors. After publishing our first paper on this research, we keep tracing the development of Econophysics and enlarge our database in time. In this paper, we will give the basic results for the network including totally 808 papers and 819 authors from 1992 to 7/30/2004.

Because the weight is a crucial factor in our network analysis, here we introduce again the measurement of weight . Based on the data set, we extracted the times of three relations between every two scientists to form a file of data recorded as 'S_1 S_2 x y z', which means author S_1 has collaborated with author S_2 'x' times, cited 'y' times of S_2' papers and thanked S_2 'z' times in all S_1's acknowledgements. One can regard this record as data of three different networks, but from the idea of transportation and development of this field, it's better to integrate all these relations into a single one by the weight of connection. Here we have to mention that in order to keep our data set closed, we only count the cited papers that have been collected by our data set and just select the people in acknowledgements which are authors in our data set.

We convert the times to weight by

$$w_{ij} = \sum_{\mu} w_{ij}^{\mu} , \tag{1}$$

in which μ can only take a value from $\{1, 2, 3\}$. So w_{ij}^{μ} is one of the three relationships—coauthor, citation or acknowledgement and is defined as

$$w_{ij}^{\mu} = \tanh\left(\alpha_{\mu} T_{ij}^{\mu}\right) , \tag{2}$$

where T_{ij}^{μ} is the time of μ relationship between i and j.

As we mentioned in the introduction; we suppose that the weight should not increase linearly, and it must reach a limitation when the times exceeds some value. So we use tanh function to describe this nonlinear effect. We also assume that the contributions to the weight from these three relations are different and they can be represented by the different values of α_{μ}. $0.7, 0.2, 0.1$ are used for $\alpha_1, \alpha_2, \alpha_3$ in this paper. The effect of different coefficients could not be revealed by any of the quantities analyzed so far.

The similarity is used here as the weight, after the network has been constructed, it is converted into dissimilarity weight as

$$\tilde{w}_{ij} = \tfrac{3}{w_{ij}} \quad (\text{if } w_{ij} \neq 0) . \tag{3}$$

It is timed by 3 because the similarity weight $w_{ij} \in [0, 3]$. Therefore, we have $\tilde{w}_{ij} \in [1, \infty]$, and it is corresponding to the "distance" between nodes. All quantities are calculated under this dissimilarity weight from now on if not mentioned.

It is interesting to compare the basic statistical results of the enlarged data set with the results given in paper [16]. Fig. 1 gives the results for degree and weight distribution in Zipf plots. The qualitative properties are unchanged, but detailed structures such as the position of a certain vertex have been changed. Fig. 2 are the vertex betweenness for two data sets. Although the qualitative properties are the same, the position of vertex has been changed. We label the positions of Stanley and Zhang as examples. In the development of Econophysics, Stanley is well-known by a series of pathbreaking works on empirical and modelling analysis of time series of economical data, such as stock prices and firm sizes, and Zhang contributed the significant step in Minority Game, an easy-understood but fruitful model for collective decision making in the economic world. These changes may reflect the development of Econophysics from the view point of network analysis. For example, we can choose a group of people working on one aspect and then tracing their

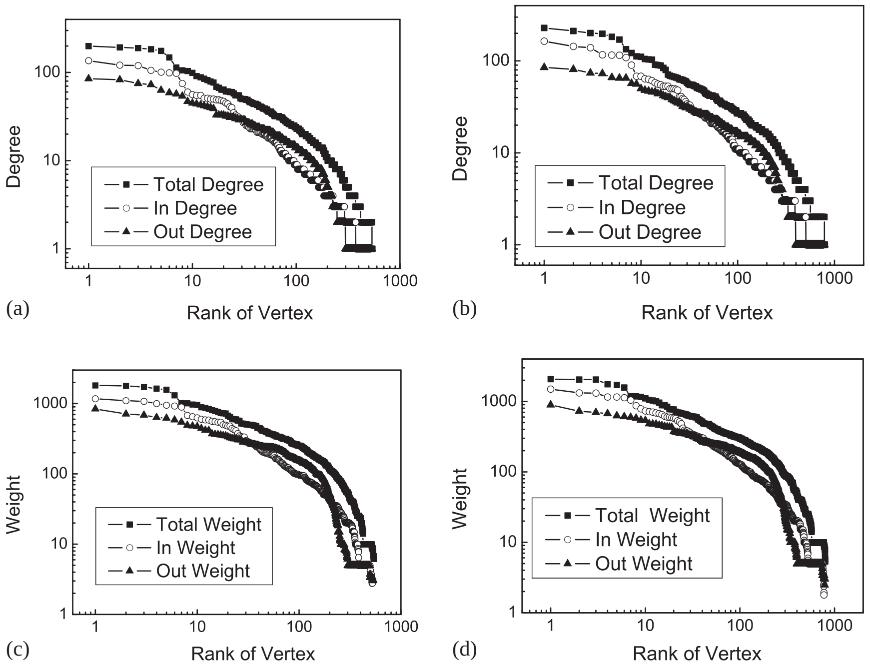

Fig. 1. Zipf plot of degree and weight for different data set. Degree distribution for 2003(a) and 2004(b). Weight distribution for 2003(c) and 2004(d).

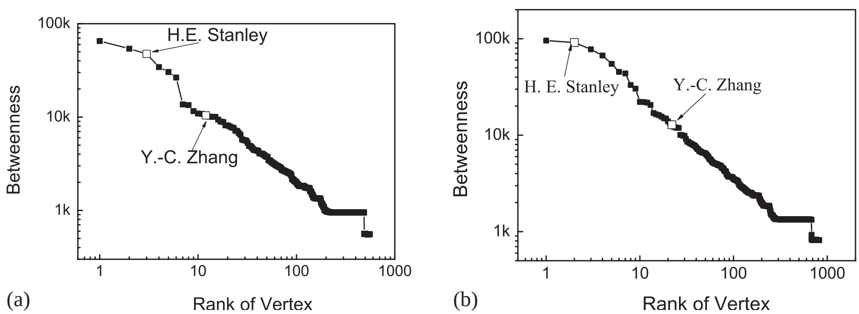

Fig. 2. Zipf plot of vertex betweenness for 2003(a) and 2004(b).

positions in the plot given above. It is easy to know the whole picture of the recent developments of this group relative to the others.

In Ref. [16], we also introduced weight per degree (WPD), a characteristic quantity of vertex. In Ref. [16], it was defined by total weight divided by total degree of every vertex. Now we will present more details of this quantities by out-WPD, in-WPD and total-WPD. Out-WPD is the quotient between the strength of outgoing relationship and the number of outgoing edges, so this represents how actively the

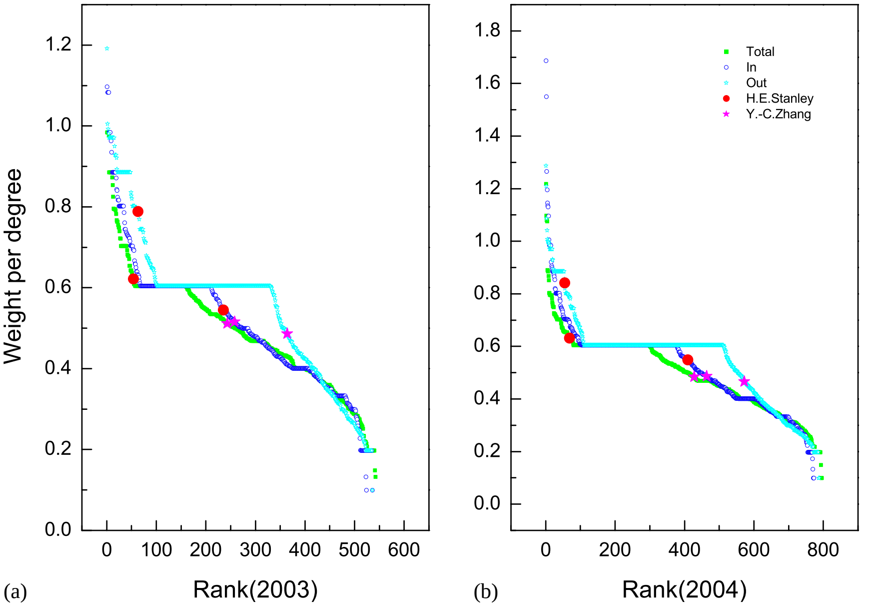

Fig. 3. Linear-scale Zipf plot of total, in and out weight per degree for 2003(a) and 2004(b). The points are marked are the WPD values of Stanley and Zhang. The platforms in all the curves suggest the working style of a large group of vertex. Weight on the platforms is about 0.604, which is roughly tanh (0.7×1). This looks like those people are connected to the community just by one cooperation.

vertex communicate with others, more intensively or more extensively. The in-WPD represents how intensively or extensively the community treated the specific vertex. For a pioneer scientist in a field, the vertex will have more edges other than more weight (times) on edges, while an evergreen vertex probably will have more weight (times) on edges other than more edges. So WPD provides a character of the working style of the vertex. For example, from Fig. 3, we can see the out-WPD of Stanley is quite large compared with the other two WPDs of him, or even compared with the out-WPD of Zhang. In some senses, this implies Stanley is a little bit more outgoing than Zhang, as the figure suggested. Any way, here we proposed this quantity only for fun, however, we wish later on it will be found some good meaning in reality such as Social Network Analysis, hopefully.

3. Clustering coefficient and the role of weight

Now we turn to the effects of weight on the structure of weighted networks. First, we introduce the way to varying the relations between weights and edges to investigate the role of weight on the structure. Then compare the different behaviors of topological quantity to reveal the effect of such variation. Especially, the clustering coefficient of weighted networks will be defined and discussed. Last, the average shortest path and betweenness will be calculated and compared.

3.1. Clustering and distance

It is well-known that the efficiency of small world network and scale-free network in real world is characterized by the coexistence of small relative distance $L(p)/L(0)$ and high-relative clustering coefficient $C(p)/C(0)$, compared with the distance $L(0)$ and cluster coefficient $C(0)$ of the induced regular network with the same number of vertex and edges. For a weighted network, a new type of random network can be introduced. The weights on edges can be randomized in weighted networks, while in non-weight networks, the only thing can be randomized is the link. This effect on the network structure is new in weighted networks, and it can be interpreted as the topological role of weight, which tells us whether the weights are distributed randomly or are related with the inherent structure.

The general approach is to change the relationship between weights and edges at a specific level p. Set $p = 1$ represents the original weighted network given by the ordered series of weights which gives the relation between weight and edge but in a decreasing order,

$$W(p = 1) = (w_{i_1j_1} = w^1 \geqslant w_{i_2j_2} = w^2 \geqslant \cdots \geqslant w_{(i_L)(j_L)} = w^L) . \tag{4}$$

$p = -1$ is defined as the inverse order as

$$W(p = -1) = (w_{i_1j_1} = w^L \leqslant \cdots \leqslant w_{(i_{L-1})(j_{L-1})} = w^2 \leqslant w_{(i_L)(j_L)} = w^1) , \tag{5}$$

which assigns the minimum weight to the edge with a maximum weight in the original network, and so on. And $p = 0$ represents a fully randomized series of $W(p = 1)$.

$$W(p = 0) = \mathit{FullyRandomized}\left(w^1, w^2, \ldots, w^L\right) . \tag{6}$$

Therefore, p in some senses behaviors like a correlation coefficient between the new and the original weight series. If we know how to generate a random series from a given series with fixed correlation p, then we can plot all the relative clustering coefficients and relative distance vs. p just like the famous figure in Ref. [19]. The way to generate a conditional random series from a given series is so-called ‘‘conditional uniform graph tests’’[22], which has more general sampling procedures to randomize a given series. However, in this paper, we only investigate the special cases corresponding to $p = 1, 0, -1$. The induced fully randomized weighted network is constructed by keeping the ordered set of edges but randomizing choosing values from the set of weights. Every edge is given a weight randomly selected from the weight set. Then we compared the basic topological properties of the original networks with the inverse or randomized one.

For a directed network, the nearest neighbor of a vertex can be defined as In, Out and Total, so the clustering coefficient of a directed network also has these three different quantities, named as *Icc*, *Occ* and *Tcc* for short. Let us take *Icc* for instance.

For every vertex v_i in the network, the vertex having edge ending at v_i forms a neighbor set ∂_i of vertex v_i. Then Icc is defined as

$$I_{cc} = \langle I_{cc_i} \rangle_i = \left\langle \frac{M_i}{|\partial_i|(|\partial_i| - 1)} \right\rangle_i , \tag{7}$$

where

$$M_i = \sum_{j,k \in \partial_i} \frac{1}{\tilde{w}_{jk}} . \tag{8}$$

This definition will give the same value as the usual clustering coefficient for directed non-weighted networks, and half of the corresponding value for undirected and non-weighted networks. The meaning of the numerator is the summation of all similarity among the neighborhood, while the denominator is the possible maximum value of similarity among them and the maximum value can be reached if everyone of the neighborhood is connected to each other and all the values of similarity are 1. So this definition has the same meaning of the clustering coefficient of non-weighted network with taking the weight of links into account.

The average shortest distance d is defined as

$$d = \frac{1}{N(N-1)} \sum_{ij} d_{ij} \tag{9}$$

in which d_{ij} is the shortest distance between vertex i,j and equals to N (the size of network) if no path exists.

The above definition is used to calculate the clustering coefficient and average shortest distance for weighted networks. Table 1 gives the clustering coefficients for the real, inverse, and randomized network respectively constructed by the data set of Econophysists and the data set of scientists collaboration provided by Newman. The later data set has the only times of collaboration between scientists, so the corresponding network is a weighted but not a directed one. The weight is given by the measurement we introduced in last section. For the fully randomized network, the result of clustering coefficient is the average of 100 random samples. In next section, the results of distribution of betweenness for fully randomized networks are also the average of 100 sampling processes. It is interesting to find that the clustering coefficient for the real network of Econophysists is obviously larger than the inverse

Table 1
Clustering coefficients of weighted network

		Real	Inverse	Random
Clustering coefficients	T_{cc}	0.064	0.029	0.038
	I_{cc}	0.057	0.033	0.037
	O_{cc}	0.067	0.015	0.029
Newman	T_{cc}	0.430	0.407	0.400

and randomized one. It is similar with situation of WS small world [19], where the randomization also leads to a gradual decrease of cluster coefficient. The difference on clustering coefficient among real, inverse and randomized networks also implies that there is certain relationship between weight and inherent network structure.

There seems to be a large difference on clustering coefficient between our network and Newman's collaboration network. The most significant reason is that the largest connected cluster dominant almost perfectly in Newman's data (83.2% of nodes are in the largest cluster), while in our case, only 25.3% nodes are in the largest cluster when the co-authorship is considered. The link in our network are dominated by the directed links for citations with smaller weight of similarity. The mean value of the weight per degree of the two network confirms such an argument: $(2.60, 1.52)$ in dissimilarity or $(0.48, 0.68)$ in similarity for our network and Newman's separately. This means the average length of our edge is much longer than Newman's, or we say, the relationship in Newman's network is much stronger than ours. This is easy to be understood because the topics covered in Newman's data is much better developed than the topics covered by our network. If we compare our clustering coefficient from largest connect cluster, 0.093, and it should times a factor of 2 because of the difference of directed and non-directed network, then it comes to 0.186. Considering further the ratio of the weight per degree of the two networks, that is $2.60/1.52 = 1.7$, the coefficient comes to 0.316. This is still smaller than 0.430, the result of Newman's network, yet comparable. Of course, it's true that there is a long way to go to develop Econophysics into a similar developed stage of Physics.

From the definition of average shortest distance expressed by formula (9), for the sparse network, the average shortest distance is dominated by the isolated vertices or small clusters, because the distance between any two disconnected vertex is set as N the size of the network. In Table 2, we give only the corresponding results for the largest connected cluster. The average shortest distance is the result of corresponding undirected cluster (if there are two directed edges between two nodes, we simply dropped the edge with smaller weight). The weight-randomized network has also smaller average shortest distance and clustering coefficient. Again, this implies the weight has some inherent relation with the structural role of edge.

In the right column of Table 2, the corresponding results for non-weighted cluster are given. We cannot compare these values with that of weighted networks. So we have compared the distribution of link and vertex betweenness for weighted and non-weighted cluster in Fig. 4. We find that the weight affects the distribution, but leads to qualitatively similar results. However, the detail according to every single

Table 2
Results for the largest cluster

	Weighted			Non-weighted
	Real	Inverse	Random	
T_{cc}	0.093	0.050	0.067	0.363
d	22.91	21.83	17.75	3.217

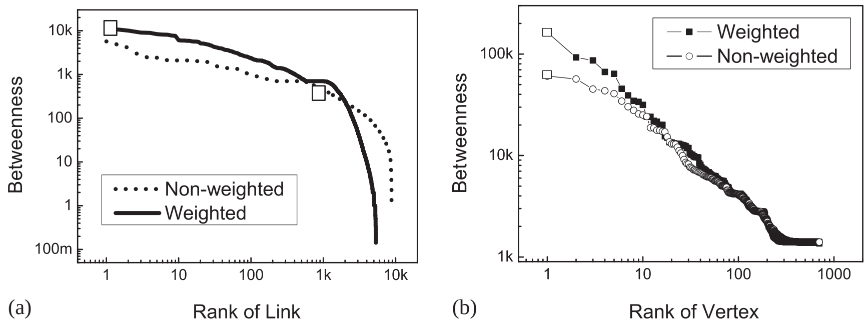

Fig. 4. Comparison of weighted and non-weighted largest cluster: (a) link betweenness, (b) vertex betweenness.

vertex is different. This is shown by the two small hollow rectangles representing the same person on the two curves. Although in the above studies we have found some effects of weight distributions, it seems that other quantities and their distributions may be needed to investigate the topological role of weight.

3.2. *Distribution of clustering coefficient and betweenness*

In order to study the impaction of weight to the topological properties of network, we have introduced the way to re-assign weights onto edges with $p = 1, 0, -1$ for weighted networks. Besides the average clustering coefficient and average shortest distance, the change of distribution of corresponding topological quantities should give more detailed descriptions for the effects of weight. Figs. 5(a) and (b) give the weight and clustering coefficient distribution for real, inverse, and randomized weighted networks. It seems that in all the cases the vertex weight distribution keeps the same forms while the distribution of clustering coefficient changes obviously.

Other important global and structural quantities of a network to investigate the impaction of weight on the structure are the vertex betweenness and link betweenness. Figs. 5(c) and (d) give the distribution of the vertex betweenness and link betweenness in Zipf plot for all cases. The distribution of link betweenness seems unchanged but if we focus on the position of a certain edge (the top one in real weighted network for instance) in the curves, it changes a lot for different cases. From the comparison here and in Fig. 4, we know the way of weight measurement affects the structure of network, but not described very well by quantities all above. We assume correlation analysis will provide more detailed information beyond this, because the measurement of weight has different effects on different quantities. For example, it does not affect the degree of vertex at all, but does affect betweenness of vertex. Maybe a correlation analysis between such quantities will tell more about the character of weighted networks. However, in Fig. 5(d), the upper tail of the distribution for the vertex betweenness has been changed, but the position of a vertex does not change as much as links. It seems that the betweenness of vertex is dominated by links more than by the weights on the links.

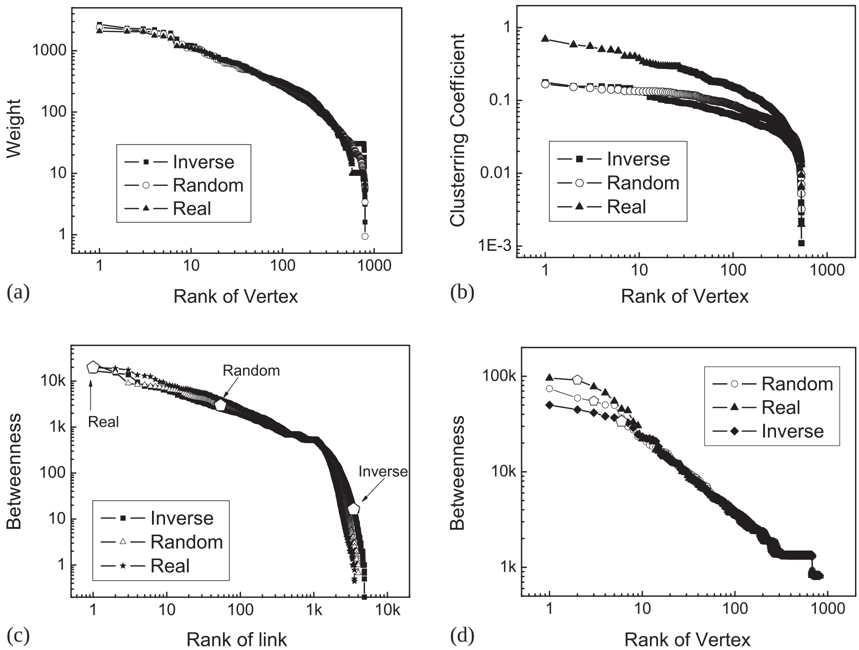

Fig. 5. Comparison of real, inverse, and randomized weighted networks: (a) vertex weight distribution, (b) clustering coefficient, (c) link betweenness, (d) vertex betweenness. All in Zipf plots.

4. Conclusions

From the comparison between networks with real, inverse, randomized weight and induced non-weight networks, we know the network structure depends on the weight. We calculated global structure quantities as clustering coefficient, betweenness of vertex and betweenness of edge under different cases. Even some global distribution seems robust but the detailed structure has been affected by the weight. These results give us clues to the question of the topological role of weight. As we point out in Section 3.1, this question investigates the relationship between weight and inherent network structure. It sounds like strong correlation existing between them. But it seems that other quantities and distributions are needed to investigate the topological role of weight. The conclusion depends on the more general exploration in more networks and modelling research.

In summary, we have constructed a small network by collecting papers in Econophysics. A new definition of weight and new topological properties are introduced and some fundamental properties are analyzed, including preliminary analysis of the topological role of weight. The idea to integrate networks with multilevel but the same kind of relationship has further application value. We wish more data can be collected including the time development of the network so that it will help to analyze the evolution of networks, especially for the networks of scientist, in which the network structure and the dynamical phenomena such as exchanging idea are in co-evolution. In this sense, network of idea transportation has some special value, because the network structure and the dynamical behavior over the network are always coupled together. So dynamical process over the network and the evolution of this network are in fact always entangled each other. As in the subway analogy, we want to extract the information about railway from traffic, but at the same time, in our network of scientists, the traffic can generate new paths!

Therefore, works on modelling such network will have very important and special value. Inspired by the empirical study in this paper, recently we have proposed a model of weighted network showing almost exactly the same behavior qualitatively. The most important character of the model is that the only dynamical variable is the times of connection, not two variables as both of connection and weight as in usual models of weighted networks. Hopefully, in the near future, we can complete the modelling work and compare the results with the empirical results here.

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